

**HYDRAULICS DIVISION**

Technical Note

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## 1.0 INTRODUCTION

A problem that occurs frequently in river engineering is how to determine high stages caused by ice during the spring breakup. Often, there is little or no information on past occurrences at the site under consideration. When this is the case, additional data may be generated based on the existing theory of river ice jams. Such data should be used with considerable discretion because application of the theoretical approach to the problem at hand is subject to many limitations, as will be explained later.

Ideally, the results of a theoretical calculation should be used as a means of cross-checking historical data of low reliability such as resident recollections, newspaper reports, archives, environmental evidence. Where historical data are completely lacking, theoretical data should be supplemented by one or two years' field observations during breakup.

In the following sections, the basis of the theory (developed by others in the past two decades) is outlined, along with methods for its practical application and a few examples. Much of the information provided herein derives from Reference 3.

A river cross section where a floating jam has formed is sketched in Figure 1. In the theory which follows, flow through the voids of the jam will be neglected and the jam assumed to be in equilibrium; the latter implies uniform flow under most of the jam's length and a longitudinal water surface slope,  $S$ , equal to the river slope under open-water conditions (14). The velocity distribution in any one vertical is taken as shown in Figure 1, where the dashed line represents the locus of the maximum velocity points; for a very wide channel, relative to its depth, the shear stress along this line is nearly zero. As a first approximation, the flows in the two subsections defined by the maximum velocity line are respectively controlled by the average shear stresses on the jam underside ( $\tau_i$ ) and on the river bed ( $\tau_b$ ).

Let  $Q_i$ ,  $A_i$ ,  $V_i$  and  $R_i$  be respectively the discharge, area, average velocity and hydraulic radius for the ice-controlled subsection. Then  $V_i = Q_i/A_i$  and  $R_i \approx A_i/W$  (see Figure 1); the Manning roughness coefficient,  $n_i$ , and the friction factor,  $f_i$ , for the jam underside are defined as  $n_i = V_i^{-1} R_i^{2/3} S^{1/2}$  (metric units) and  $f_i = 8 \tau_i / \rho V_i^2$  with  $\tau_i = \rho g R_i S$  and  $\rho$  = water density,  $g$  = acceleration of gravity =  $9.8 \text{ m/s}^2$ . Similar relationships apply to quantities pertaining to the bed-controlled flow subsection; in the following, such quantities will be designated using the same symbols as above, but with the suffix "b" in place of "i".

For the overall composite-roughness flow under the jam, designated with the suffix "o", the well-known Sabaneev equations may be used, i.e. (see References 2 and 3 for a discussion of the assumptions underlying these equations).

$$n_o = \left[ 0.5 (n_b^{3/2} + n_i^{3/2}) \right]^{2/3} \quad (1)$$

$$f_o = 0.5 (f_b + f_i) \quad (1a)$$

$$R_i/R_b = (n_i/n_b)^{3/2} \quad (2)$$

$$R_i/R_b = f_i/f_b \quad (2a)$$

where equivalent relationships in terms of the friction factors instead of the Manning coefficients are numbered as 1a, 2a, etc. Note that  $n$  and  $f$  are related by the identity

$$n = 0.113 R^{1/6} f^{1/2} \quad (3)$$

In addition to Equations 1 and 2, the following is true for wide channels:

$$R_b + R_i \simeq 2R_o = h \quad (4)$$

where  $h$  = average depth of flow under the jam; this should be distinguished from the overall depth of water,  $H$ , which is given by (assuming that the specific gravity of ice is equal to 0.92)

$$H = h + 0.92t \quad (5)$$

where  $t$  = jam thickness. To determine  $H$  and thence the jam stage,  $h$  and  $t$  must be calculated first. The next section deals with the question of determining  $t$  and  $H$ ; using the equations presented in this section,  $h$  can be calculated if  $n_o$  (or  $f_o$ ) is known and this can be determined if  $n_b$  and  $n_i$  (or  $f_b$  and  $f_i$ ) are given.

Hydraulic resistance characteristics of the river bed ( $n_b$  and  $f_b$ ) can be obtained from hydrometric surveys in the reach of interest during open-water conditions. Though jam stages are generally high, a large portion of the water depth is occupied by the jam and the river flow itself is partly controlled by the underside of the jam; thus usual values of  $R_b$  represent low open-water stages, at which  $n_b$  and  $f_b$  are stage-dependent. This dependence can generally be expressed by an empirical equation of the form:

$$n_b = a R_b^{-b} \quad (6)$$

$$f_b = a' R_b^{-b'} \quad (6a)$$

Resistance characteristics of ice jam undersides have not been widely documented to date. The only comprehensive set of field data known to the writer have been discussed and interpreted by Nezhikhovskiy (11) for freeze up jams in the

Soviet Union. Nezhikhovskiy found that the jam roughness increased with the average jam thickness,  $t$ , and he proceeded to establish empirical relationships between  $n_i$  and  $t$ . Three types of accumulations were identified, respectively comprised of loose slush, dense slush, and ice floes. The latter type is considered the most relevant to spring ice jams and the pertinent  $n_i - t$  relationship may be adopted as a first approximation for lack of better information. The writer (1) noted that, for the very rough undersides of jams,  $n_i$  should depend on  $R_i$  as well as  $t$  and attributed Nezhikhovskiy's  $n_i-t$  correlation to a restricted range of  $R_i$  values (1.0m to 1.5m). Putting  $R_i \approx \text{const} = 1.25\text{m}$ , Nezhikhovskiy's  $n_i-t$  correlation has been interpreted as follows (3):

$$f_i = 0.4 (t/R_i)^{0.8}; \text{ first alternative} \quad (7)$$

$$f_i = \left[ 1.16 + 2 \log (R_i/d_{i,84}) \right]^{-2}; \text{ second alternative} \quad (8)$$

where  $d_{i,84}$  is a measure of the absolute roughness of the jam underside, analogous to the particle diameter exceeding 84 percent of the particle sizes present on a river bed; it can be shown that the equivalent sand roughness height is equal to about  $3d_{84}$  (1). The value of  $d_{i,84}$  depends on  $t$ , as indicated in the following equation (metric units):

$$d_{i,84} = 1.43 \left[ 1 - \exp \left\{ -0.734 (t - 0.15) \right\} \right] \quad (9)$$

which has been derived empirically, as explained later. Equation 9 suggests that the jam roughness increases with  $t$  up to about  $t = 5\text{m}$  and remains constant for  $t > 5\text{m}$ .

The first alternative (Equation 7) incorporates Nezhikhovskiy's suggestion that jam roughness increases in proportion to  $t$  and assumes that  $f_i$  is an unspecified function of  $t/R_i$ . This function is then determined empirically so as to match Nezhikhovskiy's  $n_i-t$  correlation when  $R_i$  is equal to 1.25m. (Note that  $n_i$ ,  $f_i$  and  $R_i$  are related by Equation 3.) Though Nezhikhovskiy's data clearly show the jam roughness to increase with  $t$ , the suggested proportionality relationship is not beyond some doubt. The second alternative (Equations 8 and 9) makes no assumption on the roughness-thickness relationship but uses the well-known logarithmic variation of  $f$ , in analogy with established knowledge for fully

rough flow over a river bed. Using Equation 8,  $d_{i,84}$  is evaluated so as to match Nezhikhovskiy's  $n_i - t$  correlation when  $R_i = 1.25m$ ; this results in Equation 9. Data on log jams (10) show jam roughness to increase with thickness in a manner similar to that suggested by Equation 9 for ice jams. Laboratory data with plastic and ice blocks (4, 13) show a trend for  $f_i$  to increase with a measure of relative roughness, in agreement with the trend implied in both alternatives described above; for these data, "relative roughness" was represented by the ratio  $t/h$ .

Finally, it is noted that for Nezhikhovskiy's data on accumulations of ice floes, the jam thickness,  $t$ , ranges between 0.3m and 3.0m.

### 3.0 ICE JAM THEORIES

A critical review of theoretical work by others on ice jams (mainly References 10, 12, 14) has been carried out by the writer (3) and the main findings are described briefly below. Strictly speaking, the present discussion applies to spring ice jams; these may be considered granular accumulations of ice fragments with no cohesion. It is not clear at this time whether cohesionless jams form during freeze up, due to complications arising from thermal effects. From the practical point of view, a cohesionless jam is of more interest as it has a greater flooding potential than a cohesive jam.

For most natural streams (width-to-depth ratio larger than about 10), the thickness of a spring jam is controlled by its ability to resist streamwise forces caused by its own weight and the shear stress exerted by the flow on its underside. This principle, combined with hydraulic resistance considerations discussed earlier, results in

$$\eta \equiv H/WS = 0.63 f_o^{1/3} \xi + \frac{5.75}{\mu} \left\{ 1 + \sqrt{1 + 0.11 \mu f_o^{1/3} \left(\frac{f_i}{f_o}\right) \xi} \right\} \quad (10)$$

in which  $H$  = overall water depth due to the jam, as defined in Equation 5 and  $\eta$  is a dimensionless version of  $H^*$ ;  $\mu$  = dimensionless coefficient that depends on the internal friction of the jam; and  $\xi$  = dimensionless stream discharge, defined as

$$\xi \equiv (q^2/gS)^{1/3} / WS = y_c / WS^{4/3} \quad (11)$$

where  $q = Q/W$  and  $y_c$  = critical flow depth. The first term on the RHS of Equation 10 represents the dimensionless depth of flow under the jam,  $h/WS$ , and the second term gives the dimensionless submerged thickness of the jam,  $0.92 t/WS$ . Note that Equation 10 gives  $\eta > 0$  for  $\xi = 0$  which is implausible and probably due to the fact that the theory does not allow for grounding of a jam that may occur at very low discharges.

To test Equation 10, data from recent documentations of ice jams known to have been floating and in equilibrium (1, 2, 5, 6, 7, 8) have been used to plot  $\eta$  vs.  $\xi$ , as shown in Figure 2. With considerable scatter, the data points

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\* The dimensionless parameters appearing in Equation 10 have been chosen for convenience in interpreting field data and comparing Equation 10 with other theories (3).

show  $\eta$  to increase with  $\xi$ . In addition to observational errors, the scatter in Figure 2 could be due to variations in the parameters  $\mu$ ,  $f_o$  and  $f_i/f_o$ . From detailed analysis (2) of the case studies represented in Figure 2, average values of these parameters have been estimated as  $\mu = 1.2$ ,  $f_o = 0.45$  and  $f_i/f_o = 1.25$ . With these values, Equation 10 reduces to

$$\eta = 0.48 \xi + 4.8 (1 + \sqrt{1 + 0.13 \xi}) \quad (12)$$

which is also plotted in Figure 2 and seen to provide a fair description of the data points. It appears that the theory of floating equilibrium jams is basically sound.

Finally, it is noted that the above described theory applies to so-called "wide channel" jams. There can exist another type of jam, the so-called "narrow channel" jam of which the thickness is controlled by the no-submergence condition of the leading edge (12). The "narrow" type of jam is not as severe as the "wide" variety and is not expected to form during breakup in streams with width-to-depth ratios over about 10 (3). The "narrow" jam could conceivably form in wider channels during freeze up if thermal effects make it stronger than would be expected from internal friction effects alone.

#### 4.0 METHODS OF PRACTICAL APPLICATION

Before considering methods of applying the above results to practical problems, it is advisable to enumerate the limitations of the theory.

- (i) The theory assumes a very wide rectangular prismatic channel. Application to rivers implies that a natural stream may, for the purpose of ice jam calculations, be replaced by a rectangular prismatic channel of equivalent average dimensions.
- (ii) The theory applies to floating jams in equilibrium and gives the jam stage assuming that a jam has formed, has reached equilibrium, and fully affects the location of interest. In reality, one or more of these conditions may not be satisfied during a given breakup period. It follows that a theoretically derived jam stage - discharge relationship can only provide an upper envelope of actual events, barring the occurrence of severe grounded jams about which no theory is available.
- (iii) Theoretical prediction depends partly on the hydraulic resistance of a jam's underside which has been evaluated on the basis of only one set of field data.
- (iv) The theory does not take into account special constraints that may be present such as existence of low flood plains, by-pass channels, possible effects of bridge piers on jamming frequency, etc. The possible effects of such features require assessment by careful inspection of the site of interest.

With these qualifications, let it be assumed that a jam stage-discharge curve is to be generated for a reach about which the following information is given: channel slope; open-water rating curve (stage vs. discharge); reach-average flow area and reach-average water surface width versus stage. From this information, the relationships  $R_b$  (reach-average open-water flow depth) vs. stage and  $f_b$  vs.  $R_b$  can be derived. The latter can be conveniently quantified for the anticipated range of  $R_b$  values under the jam, as suggested by Equation 6 (or 6a). From considerations outlined in the previous sections, two methods of calculation seem to be possible, a detailed method and a simplified method.

#### 4.1 Detailed Method

After manipulation of the equations stated so far, the following procedure is suggested.

- (i) Assume a value of  $t$  and compute  $R_i$  from

$$R_i = t \left[ (\mu t / 13.6 \text{ WS}) - 0.92 \right] \quad (13)$$

which is a rearranged version of the equation expressing the balance of forces within the jam (1). Use  $\mu = 1.2$  unless there exists evidence in favour of a different value.

- (ii) Compute  $f_i$ ; this may be done either by Equation 7 (alternative 1a) or by Equations 8 and 9 (alternative 1b).  
 (iii) Compute  $Q$  from

$$Q = W \sqrt{8gS} R_i \sqrt{R_i/f_i} \left\{ 1 + \frac{a' \frac{1}{1+b'}}{f_i} \left( \frac{f_i}{R_i} \right)^{\frac{b'}{1+b'}} \right\} \quad (14)$$

- (iv) Compute  $R_b$  from

$$R_b = (a' R_i / f_i)^{\frac{1}{1+b'}} \quad (15)$$

- (v) Compute  $h = R_b + R_i$ ; enter open-water stage versus  $R_b$  graph with  $h$  in place of  $R_b$  and find the stage that corresponds to the bottom of the jam; add  $0.92t$  to find the jam stage for the discharge computed in step (iii).  
 (vi) Repeat for a few values of  $t$  and plot jam stage vs.  $Q$ .

This approach implicitly assumes that the river width  $W$  does not change with stage which is a fair approximation for many streams. In cases where  $W$  changes appreciably with stage, the above procedure may be modified to carry out the computation as a trial-and-error process. Note that  $W$  is defined as the width at the level of the bottom of the jam.

#### 4.2 Simplified Method

This involves direct use of Equation 12 which is a "calibrated" version of the theory, based on average values of  $\mu$ ,  $f_i$ , and  $f_i/f_o$ . Calculation may be done according to the following steps.

- (i) For a given  $Q$ , compute  $\xi$  (Equation 11).  
 (ii) Compute  $h$  from

$$h = 0.48 \xi \text{ WS} \quad (16)$$

- (iii) Enter open-water stage vs.  $R_b$  graph with  $h$  in place of  $R_b$  to find the stage corresponding to the bottom of the jam.
- (iv) To this stage, add  $0.92t$ , as computed from

$$0.92t = 4.8 WS (1 + \sqrt{1 + 0.13\xi}) \quad (17)$$

## 5.0 EXAMPLES

Figures 3, 4 and 5 give the results of calculations by the above methods, for the following sites:

- Peace River at the town of Peace River (Figure 3)
- Smoky River at the town of Watino (Figure 4)
- Thames River at the town of Thamesville (Figure 5)

The data points in Figures 3, 4 and 5 are actual peak stages during both freeze up and breakup as determined from the following sources: Water Survey of Canada (WSC) gauge records; Alberta Environment; B. C. Hydro; writer's own observations at Peace River and Watino. At these locations, the WSC recording gauges often malfunction during breakup due to ice damage; in such instances, stage may be measured using manual WSC gauges that are mounted on nearby bridge superstructures. In general, ice conditions associated with the data points of Figures 3 to 5 are unknown. Where simultaneous visual observations are available, pertinent notes are shown beside the corresponding points.

Reach-average hydraulic parameters under open-water conditions were obtained from Kellerhals et al. (9) for Peace River at Peace River and Smoky River at Watino; and from a hydrometric survey of the Thames River at Thamesville carried out by the writer in June 1980. For the Peace and Smoky calculations, constant widths were assumed. For the Thames, however, it was necessary to allow  $W$  to vary and apply a trial-and-error calculation procedure. The ranges of the dimensionless discharge parameter,  $\xi$ , for the breakup peaks shown in Figures 3 to 5 are 34-115 (Peace River), 35-84 (Smoky River) and 920-1290 (Thames River).

The open-water rating curves of the gauges corresponding to Figures 3 to 5 are also shown for convenience in assessing the effects of ice on stage. Note that these curves provide lower envelopes of the data points.

The largest discrepancy between data and theoretical calculation seems to occur for Peace River at Peace River. It is likely that significant equilibrium jams do not occur frequently near this site, and this coincides with the writer's field observations for the years 1974, 75, 76 and 79. In 1974, a secondary stage peak occurred at Peace River, caused by a major jam known to have formed about 15 km downstream. Allowing for the channel slope, it is

estimated that this jam would have produced a stage of 322m\*, had it formed near the gauge. A jam believed to have reached equilibrium formed at Watino in 1977. Figure 4 shows that the stage of this jam is about 1m less than that predicted by the detailed method-alternative 1b. The 1976 peak at Watino was estimated from a post-breakup survey of high water marks and appears to have been 0.7m higher than the value predicted theoretically. Figure 5 shows the best agreement between theory (detailed method) and data. It is perhaps significant that gauge malfunction due to ice damage is rare at Thamesville owing to relatively deep, tranquil flow and thin ice cover.

Of the three calculation methods applied herein, the simplified method gives the worst results while alternative 1b of the detailed method gives somewhat better results than does alternative 1a. In general, the deviation between theory and actual peak stages increases with increasing discharge. This is considered reasonable since the larger the discharge, the lesser the probability that an ice jam will form and attain equilibrium.

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\* It is improbable that such a stage can actually occur at Peace River since the crest of the existing flood protection dykes has an elevation of 319.4m.

**SUMMARY AND CONCLUSIONS**

Methods for practical application of the existing theory of river ice jams have been presented and tentatively evaluated by means of three examples. The theory is subject to several limitations and should only be applied where historical data of high reliability are lacking. The theory does not provide a means for assigning frequencies to predicted stages; it simply predicts the stage due to a floating equilibrium jam, assuming that such a jam has formed. Hence, the theoretical calculation would be best utilized as a means of cross-checking historical data of low reliability. Where historical data are completely lacking, theoretical data should, as much as possible, be supplemented by one or two years' field observations. Special constraints that may apply to the site of interest, e. g. low flood plains, by-pass channels, etc., should be identified and their possible effects assessed by careful site inspections.

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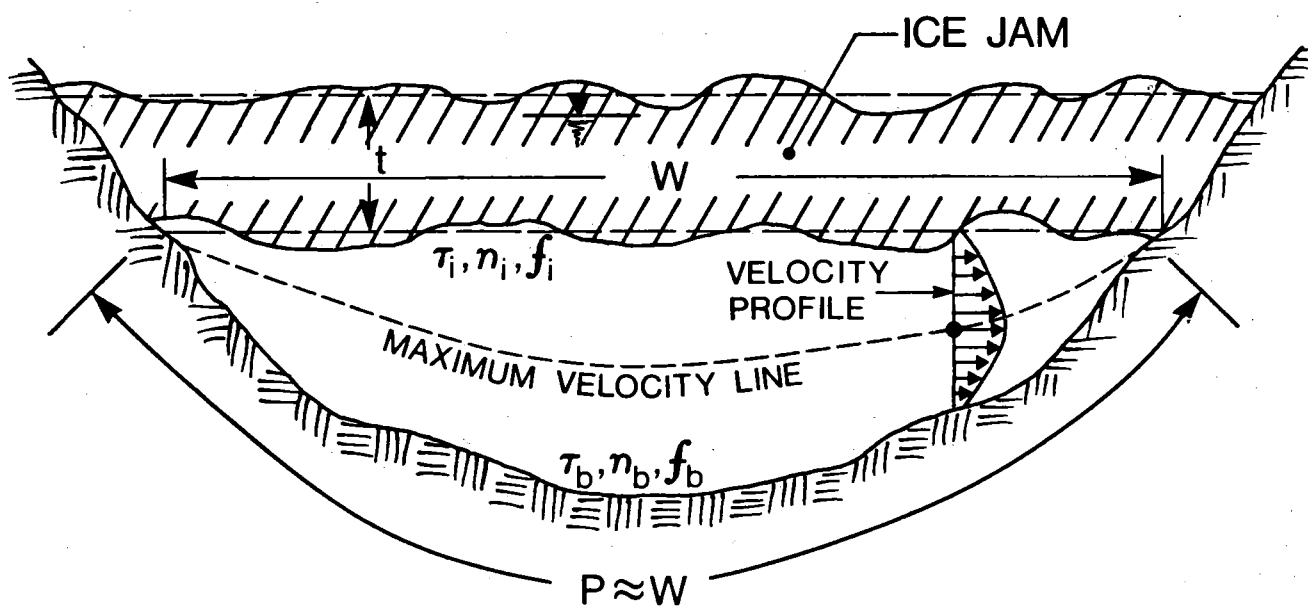


Fig. 1 River cross section with a floating jam; definition sketch

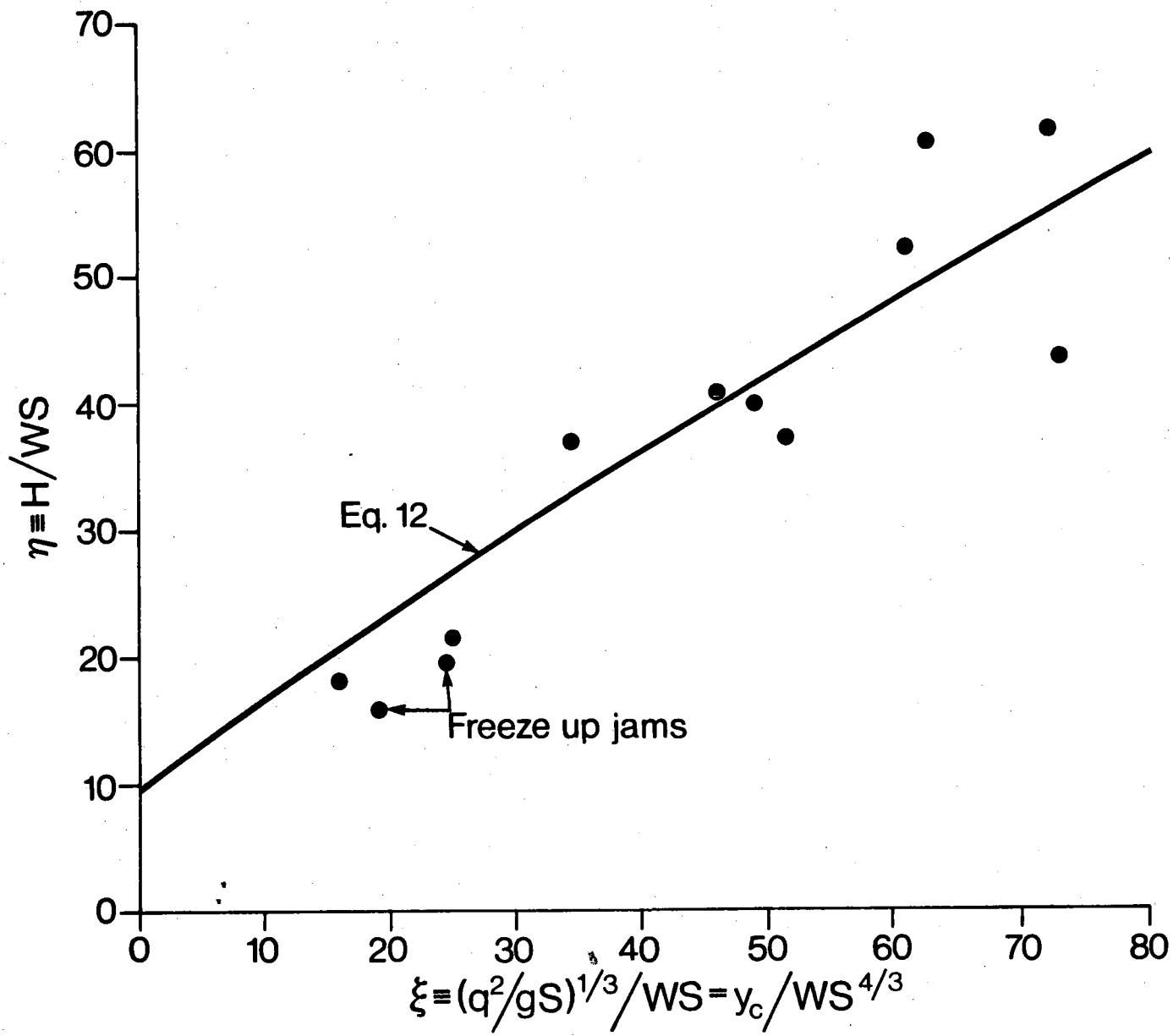


Fig. 2 Dimensionless jam stage,  $\eta$ , versus dimensionless discharge,  $\xi$

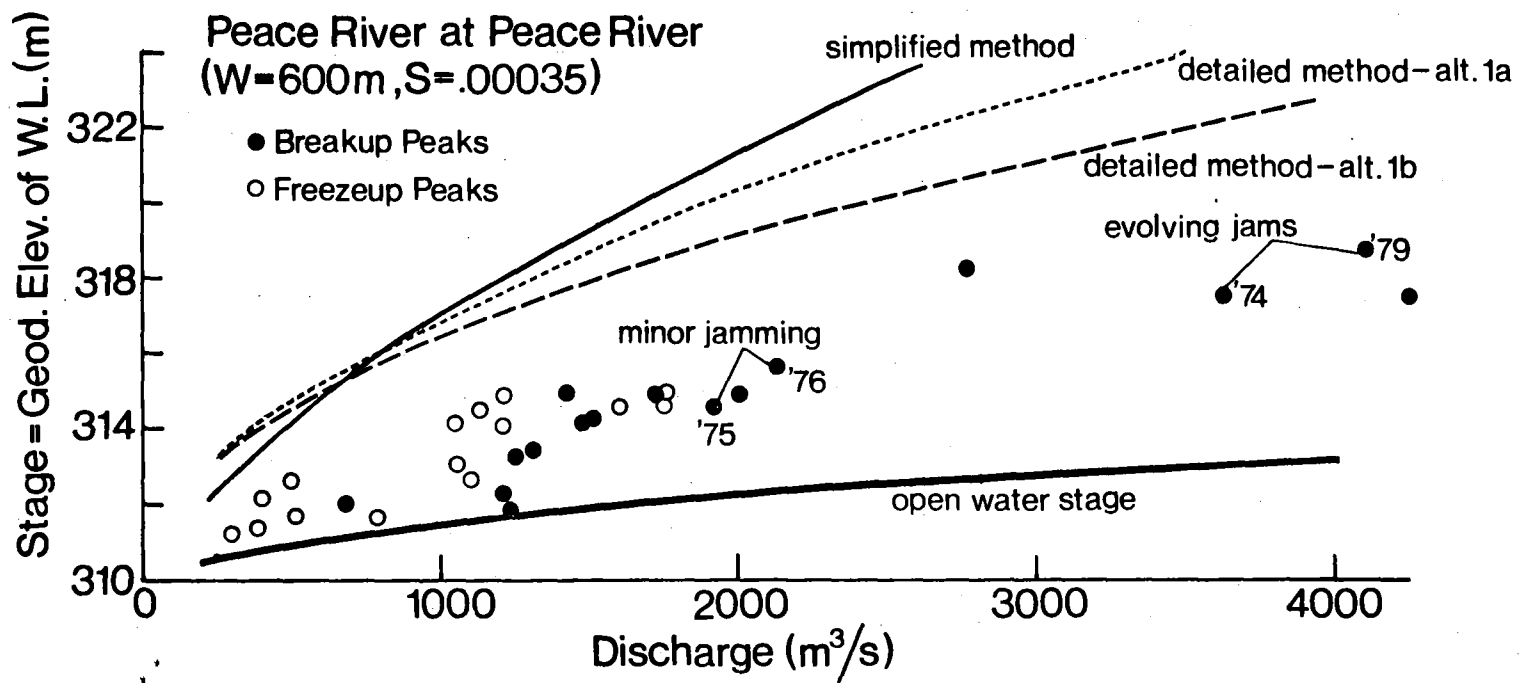


Fig. 3 Historical ice-influenced peak stages versus discharge - Peace R. at Peace River

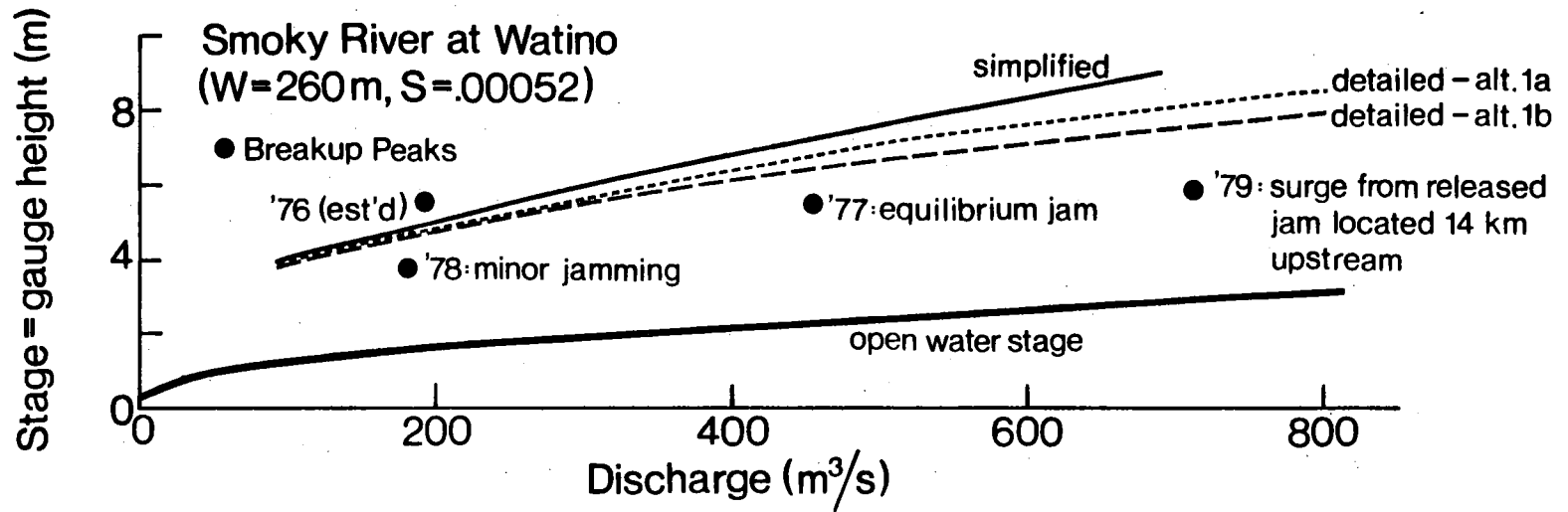


Fig. 4 Observed peak breakup stages versus discharge - Smoky R. at Watino

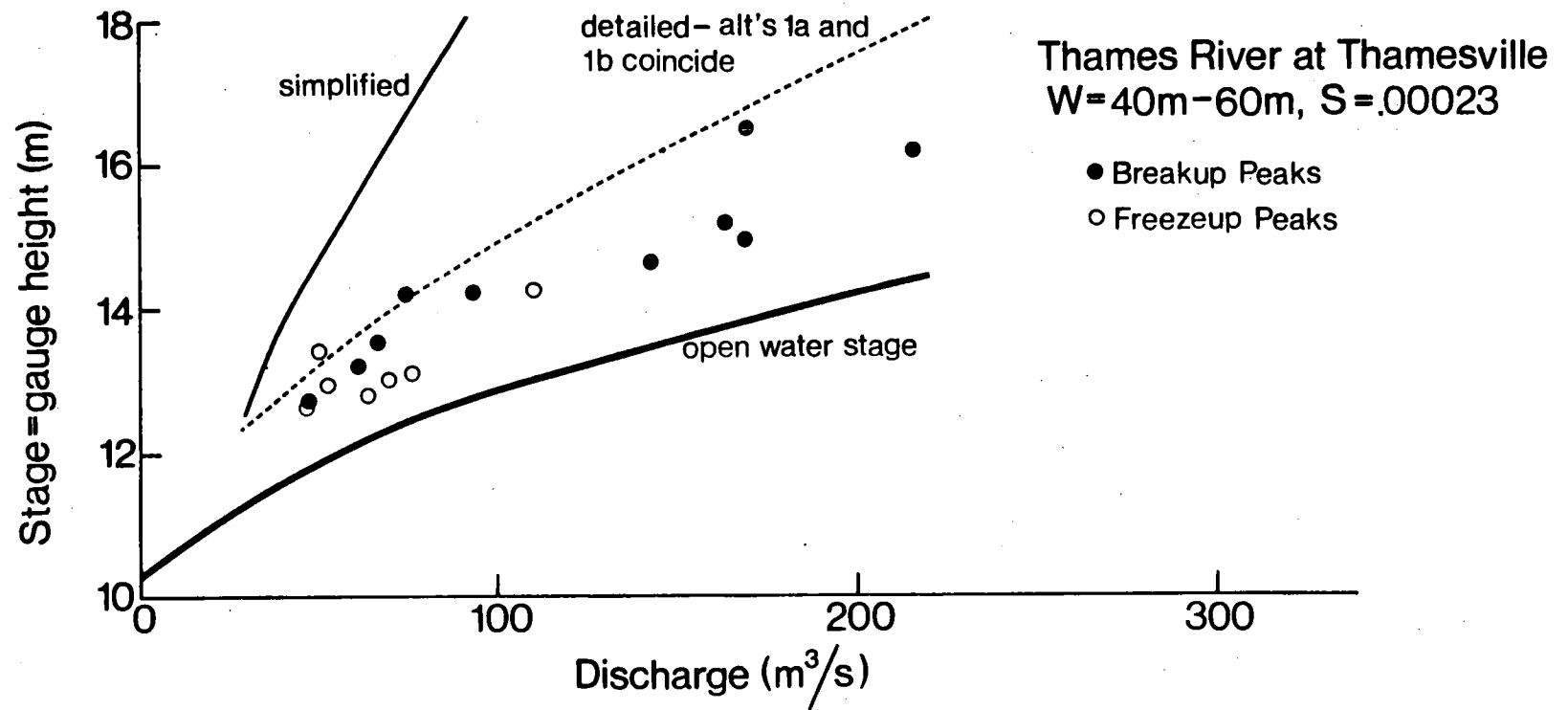


Fig. 5 Historical ice-influenced peak stages versus discharge - Thames R. at Thamesville

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Methods to predict ice jam stages in  
rivers.

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Beltoas, S.  
Methods to predict ice jam stages in  
rivers.

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