Kalman Filter and Satellite Attitude Control System Analytical Design*

Y.V. Kim

Canadian Space Agency, 6767-route de l'Aeroport, St-Hubert, J3Y 8Y9
Quebec, Canada (Tel: 613-990-7407; e-mail: yuri.kim@ canada.ca).

Abstract: Certain robust form of suboptimal Kalman-Busy Filter (KBF), with Bounded Grows of Memory (FBGM) is considered for satellite Attitude Control System (ACS) design purposes. At the first step of the design, at the preliminary (conceptual model – CM or low fidelity model - LFM) design it can be used to synthesize the system state estimator/observer and the controller and to evaluate system feasibility and potentially achievable performance. Further steps of the design, take into account some realistic constrains and restrictions, and develop this LFM, by adding more detail elements, to complex, non-linear and not-stationary high fidelity model (HFM). This model (HFM) should be analyzed from the System Engineering point of view, during design review, prototyping, manufacturing, testing and qualification. However, the CM, as the basic reference, still keeps its significance to the end of the system life time cycle.
If the conceptual design was performed professionally by the experienced developer, it is usually sound and robust, and the LFM can be considered as a generic model that would not be drastically changed developing the HFM model. Rather some additional elements and constrains would be introduced, but not making unfit previous analysis and the results.
More than that, with small additions synthesized at the conceptual design phase state estimator and controller can be directly implemented in the Flight System. However, even experienced specialist is not assured against mistakes and misinterpretations sometimes leading to wrong solutions. Also new developers (beginners) can be involved, students should have a solid methodology based on an objective criteria rather than on the tutor experience and intuition. All this presents the actuality of the consideration presented below. It is in the scope of the linear, time invariant systems under assumptions of the linear KBF theory. If these assumptions are applicable then presented results can be implemented or, at least used as a reference, to guide developers through the system developing process.
Keywords: analytical design, dynamic system, satellite attitude control system (ACS), Kalman Filter, controller, estimator, filter, white noise, optimal estimation and control.
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Biographical notes:

Y.V.Kim graduated from Moscow Aviation Institute as an Electro-Mechanic Engineer in Aerocpace GN&C and Avionics. Ater graduation, he worked in the Aerospace Industry and universities of the USSR as an engineer, researcher, scientist, manager, professor. He obtained his PhD and Doctor of Technical Science degrees in Aerospace GN&C. After the colapce of USSR, he worked for Israel Aeropspace Industries Corporation in Israel and Canadian Space industry in Canada as an Engineer and Scientist, lectured and trained Canadian students and new engineers and scientists. His main areas of professional inerest are state estimation and multisensory navigation systems, his more resent works has been in new methods for satellite navigation and control. He has many scientific publications and inventions that have been implemented in flight sytems in USSR, Israel and Canada.

*Note: Analytical Control Design aka Optimal Controller Synthesis is Russian terminology for Optimal feedback control (American)

1. Introduction

Analytical Design (AD) is the mathematical system consideration that can be used at the Conceptual Design stage (Technical Proposal) to present the *Conceptual system model* (CM) and its potentially available performance, if some presumable assumptions would be accepted and negligible at this phase engineering constrains and restrictions are removed. For the closed loop Automatic Control System AD is *the synthesis* of system estimator and controller that is based on some optimal (minimization) criteria.

CM can also be used at any further stage of system developing (HFM) to check if the considered system functionality and performance can be expected as it was put in the system concept. Of course, the necessary condition for this presumes the conceptual heritage in further developing stages that is based on keeping conceptual developers in the loop of system developing cycle and certain technological discipline, preventing against spontaneous changes due any subjective factors. The AD approach is well combined with the Model Based Design (MBD) (R. Aarenstrap, 2015) that is using of system mathematical models and simulation through the end-to-end developing process and is currently changing System Engineering technology all over the world.

System conceptual model (CM) can be developed at AD stage being based on the Applied Optimal Control and Estimation Theory (Kwakernaak & Sivan, 1972; Braison &Yu-Chi Ho,1975) and to be considered as the optimal premature assessment.

Uncertainties about the actual system architecture, components and disturbances may not be very critical at this stage, if a mature sophisticated developer/expert performs the AD basing on his accumulated engineering experience, skill and intuition. From the other side the assessment cannot be made entirely empirically being based on the past experience and intuition. Some objective criteria should be used. Such criterion, the extremum of system performance quality functional was used in the fundamental works of R. Bellman (Dynamic Programming, extremum minimum, 1957), L. Pontriagin (Principe of maximum,1976) to find optimal controller and R. Kalman (State-space and linear system estimation, 1960), R. Kalman and R. Bucy (1961) to find optimal estimator. These works served as the base for the development of the applied methods providing powerful mathematical tools to optimize estimation and control processes for a wide class of different systems , for different applications. For the linear stochastic systems, affected by the white Gaussian noises, with a quadratic functional used as the optimization criterion, were found optimal linear negative feedback controller (LTI-OC) and linear optimal estimator (LTI-OE: Kalman-Bucy Filter-KBF (aka Kalman Filter-KF for the discrete systems), with coefficients obtained by solving similar matrix Riccati equations that determine system optimal control quality criteria matrix S and the optimal estimation error covariance matrix P .

The Separation Theorem was also formulated stating that the optimal estimation and control system is the LTI-OE and the LTI-OC, consequently connected in the negative feedback closed control loop that could be designed separately, however, providing system stability.

These important for the practical application results can be found in the following publications: Bryson & Yu-Chi Ho (1975), Kwakernaak & Sivan (1972), Gelb *et all* (1974), F.L. Chernousko, V.B. Kolmanovskiy (1978).

Analytical design of the Aerospace control and navigation systems started in 70-th in former USSR with the purpose to find optimal dynamic for some conventional systems used in practical applications; such as Autopilots, Inertial Navigation system, Launch Rocket Control system and others. Some examples were published by the following authors: Krasovsky (1973), Letov

(1979*)*, Kim & Nazarov (1975). Last results, in this area, known to author, were dedicated to the optimal orbital control, for satellites flying in formation with KF. They were published by Vuckovich & Kim in (2015).
The aim of this paper is to present more resent results of the development of special modification of Kalman filer (KBF), the Filter with Bounded Grows of Memory (FBGM) (Y.Kim, 1990, Y.Kim, P. Kobzov, 1991, Y.Kim, 2008) and using it for the synthesis of satellite feedback control, closed control loop (ACS). Also it aims to draw developers attention to AD, that it can be a useful stage in system development helping to predict expected system performance and verify it in various tests.
It is well known that in many cases such conventional engineering approach as using linear time-invariant system (LTI) with constant control and estimation gates, is a sound and robust design that can solve the control problem enough successfully. Hence, in many cases there is no need to put additional computational load on on-board computer (OBC) for implementing rigorously non-linear and non-stationary optimal estimation and control equations to solve them numerically in OBC in real time. More than this, following by this way, in many instances due to inappropriate statistical data, used for rigorously optimal design, in practice designer can get unstable behaviour and malfunctioned system.
The approach of multisensory measuring system (MMS) - estimation and correction of measured errors, that is conventional for an airplane, is demonstrated for satellite attitude and angular velocity estimation problem. It allows for designing measuring multisensory subsystem separately and independently of the controller, delegating to each of them only specific for them tasks; MMS-reducing measured errors and controller-satellite body frame alignment and counteraction to applied disturbing torques.

1 Kalman-Busy Filter sub-optimization

Since publication the Kalman–Bucy Filter (KBF) (1961, Kalman & Busy) has become a very popular mathematical tool for solving diverse applied estimation problems. The filter provides optimal estimates for all observable variables of the linear stochastic dynamical system that complies with the KBF theory assumptions. This filter is presented by the matrix equations as follows below. Let us given linear, fully observable (Y.Kim, G. Gonchrenko, 1981) and controllable stochastic system

$$\begin{aligned} \dot{x} &= Fx + Gw, \\ z &= Hx + v, \end{aligned} \qquad (1.1)$$

where: x is an n -vector of the system state, w is an m -vector of external disturbances, z is a p -vector of measurements, v is a p -vector measurement noise, is an $(n \times n)$ system dynamics matrix, G is an $(n \times m)$ disturbances matrix, H is a $(p \times n)$ measurements matrix.
Let us the following information about (1) is given:
F, G, H are known matrices of time , these in the stationary case are matrix constants and

$$
\begin{aligned}
&E[x(t_0)]=0,\ \ E[w(t)]=0,\ \ E[v(t)]=0,\\
&E[x(t_0)x^T(t_0)]=P_0,\\
&E[w(t)v^T(\tau)]=E[v(t)w^T(\tau)]=E[w(t)x^T(t_0)]=0,\\
&E[w(t)w^T(\tau)]=Q(t)\delta(t-\tau),\\
&E[v(t)v^T(\tau)]=R(t)\delta(t-\tau),
\end{aligned}
\tag{1.2}
$$

where: P_0 is the initial state covariance matrix, $R(t)$ is the covariance matrix of measurement noise, $Q(t)$ is the covariance matrix of disturbance noise, $\delta(t-\tau)$ is the Dirac delta function. Hence, $w(t)$ and $v(t)$ are Gauss white noise processes. Usually, matrixes Q and R are diagonal and in the stationary case, these matrices are constants, having practically meaning of the spectral densities of the white noises $w(t)$ and $v(t)$, correspondingly

$$
\begin{aligned}
Q = S_w = diag\left[2\sigma_{wi}^2 T_{wi}\right],\ \ {}_{i=1,2,..m}\\
R = S_v = diag\left[2\sigma_{vj}^2 T_{vj}\right],\ \ {}_{j=1,2,..m}
\end{aligned}
\tag{1.3}
$$

where σ_{wi}, T_{wi} and σ_{vj}, T_{vj} are the standard deviations (STD) and the correlation times (Tc) of these stochastic processes w and v that idealistically are presented in KBF theory like the white noises with zero Tc and infinite STD.

This system state (state vector x) at any time instant t can be estimated (found the optimal estimate $\hat{x}$ for x) with providing the minimum of system state estimation error ($\tilde{x}=\hat{x}-x$) for the covariance matrix $P = E\left[\tilde{x}(t)\cdot\tilde{x}^T(t)\right]$ diagonal $J = diagP$. In other words, KBF filter merit criterion is as follows

$$
J_{\min} = \min(diagP) \tag{1.4}
$$

This criterion is provided when the KBF is used for estimation of (1.1). The KBF is usually presented by the following matrix equations

$$
\begin{aligned}
&\dot{\hat{x}} = F\hat{x} + K(z - H\hat{x}),\ \ \hat{x}_0,\\
&K = PH^T R^{-1},\\
&\dot{P} = FP + PF^T - PH^T R^{-1} HP + GQG^T, P_0
\end{aligned}
\tag{1.5}
$$

where: $K = K(t)$ is the KBF weigh matrix gain, $P = P(t)$ is the KBF estimate errors covariance matrix that can be found from the solution of the third matrix equation in (1.5) (Riccati type equation). Equation (1.5) presumes that measurement vector z is continuously available, and then (1.5) presents KBF in the , so called, “filtering mode”. However, if at some time instance t_p the measurement process is ended or temporarily interrupted and the vector measurements z since then is not available, then the KBF can be transitioned in the “prediction” mode. This mode is obtained from the filtering mode by setting in (1.5) KBF matrix gain to zero

$K(t) \equiv 0,$ when $t \geq t_p$

$$
\begin{aligned}
&\dot{\hat{x}} = F\hat{x}, \quad \hat{x}_0 = \hat{x}_p, \\
&K \equiv 0, \qquad\qquad\qquad\qquad (1.6)\\
&\dot{P} = FP + PF^T, P_0 = P_p
\end{aligned}
$$

To implement KBF in practice, specifically in real time on-board computer (OBC) with limited computational capabilities, it is always useful to sub-optimize the filter, sacrificing potentially achievable maximal accuracy, for the simplicity and the robustness (*quid pro quo*), being satisfied by some tolerate level of (1.4) instead of exact minimum at any considered time instance.

As author presented in the past publications (1990, Kim, 1991, Kim & Kobzov, 2009, Kim), the KBF in the time domain can be equivalently decomposed into two filters, working in parallel; non-stationary KBF with time variant matrix coefficient $\tilde{K}(t)$ and stationary KBF with time invariant (constant) coefficient K^*, that can be determined in advance, before using the KBF for any purposes.

Both coefficients can be determined with the solution of modified KBF Riccati matrix equations for covariance matrixes $P^{\sim}(t)$ and P^*. This KBF modification is as follows

$$
\begin{cases}
\dot{\hat{x}}^{\sim} = F^*\hat{x}^{\sim} + \tilde{K}(z - H\hat{x}^{\sim}), \quad \hat{x}_0^{\sim}, \\
\tilde{K} = \tilde{P}H^TR^{-1}, \\
\dot{\tilde{P}} = F^*\tilde{P} + \tilde{P}F^* - \tilde{P}H^TR^{-1}H\tilde{P}, \ \tilde{P}_0 = P_0 - P^*, \\
F^* = F - K^*H, \\
\dot{\hat{x}}^* = F\hat{x}^* + K^*(z - H\hat{x}^*), \ \hat{x}_0^*, \\
K^* = P^*H^TR^{-1}, \\
FP^* + P^*F^T - P^*H^TR^{-1}HP^* + GQG^T = 0 \\
\hat{x} = \hat{x}^{\sim} + \hat{x}^*, \ \hat{x}^{\sim}(t \to \infty) \to 0, \ \hat{x}(t \to \infty) \to x^* \\
P = \tilde{P} + P^*, \ \tilde{P}(t \to \infty) \to 0, \ P(t \to \infty) \to P^*
\end{cases}
\qquad (1.7)
$$

Original Riccati equation for the matrix P is split here into two equivalent equations: the differential equation for the transfer state $\tilde{P}$ (3-d of (1.7)) and the algebraic one (7-th of (1.7)) for the steady state P^*. For many practical applications takes place the following inequality

$$
diagP_0 >> diagP^* \qquad (1.8)
$$

In this case, as it was showed in (Kim, 1990, pp.109-112), two parallel KBF filters (1.7) can be approximately substituted by the single suboptimal filter, working consequently in time in two modes: at the beginning in "initial filtering" (IF) mode for the "quasi- deterministic" system (assuming that in (1) $w = 0$ and $Q = 0$) with time-variant (variable) gain $\tilde{K}(t)$ gate and after is automatically switched to "steady filtering" (SF) for the substantially stochastic system model with time-invariant (constant) gain K^* (assuming that after some IF period t_*, $t \geq t_*$, the transfer process practically entirely completed and SF can start). Unlike the KBF, that is a filter with unbounded grooving memory (assuming continuous solving of KBF Riccati equation to determine KBF gate taking into account for it all process prehistory), this suboptimal modification was named the "Filter with Bounded Growing Memory (FBGM). The filter equations are presented by (1.9), (1.10) below

$$\dot{\hat{x}} = F\hat{x} + K(z - H\hat{x}),$$
$$K = \begin{cases} \tilde{K}, t_0 \leq t \leq t_*, \\ K^*, t > t_*, \\ 0, \quad if \quad z \equiv 0, \end{cases} \tag{1.9}$$

where: t_* is the time, required for unbiased estimation of all r components of vector x, when covariance matrix $\tilde{P}$ decaying to a small matrix $\tilde{P}(t_*) \approx 0$. The gains $\tilde{K}$ and K^* are found with the following formulas

$$\begin{cases} \tilde{K} = \tilde{P}H^T R^{-1}, \\ \dot{\tilde{P}} = F\tilde{P} + \tilde{P}F^T - \tilde{P}H^T R^{-1} H\tilde{P}, \tilde{P}_0 = P_0, \\ K^* = P^* H^T R^{-1}, \\ FP^* + P^* F^T - P^* H^T R^{-1} HP^* + GQG^T = 0. \end{cases} \tag{1.10}$$

In other words, $\tilde{K}$ is computed for system (1), considered as a "quasi- deterministic" ($w \equiv 0, Q = 0$, only transfer process, caused by x_0 takes place), and K^* - considering (1) as a "substantially- stochastic" system, where only steady state motion, caused by the random disturbance w takes place, however the transfer process has been approximately decayed.

Three modes can be considered for the FBGM: IF when $K = \tilde{K}$, SF when $K = K^*$, and the "prediction mode" (P) when measurement vector z is not available and $K = 0$.
This filer can be easily periodically restarted after any interruption (outage) in measuring process. In the articles (Kim, 1998 p.228) was introduced the "*observability index*" χ_i (power of signal to power of noise ratio) for certain $i-th$ component of the signal measured from the estimated system (1), considered as a quasi-deterministic. At the transition time t_* when KBF is switched from the IF mode to the SF mode all the observability indexes χ_i become bigger

than one ($\chi_i >> 1,\ i = 1,2,...n$). This index is helpful to use it for FBGM analysis in the IF mode.

For filter analysis in the SF mode also can be introduced special *"filterability index"* $\xi_i = \frac{q_j}{r_i},\ j = 1,2,..m;\ i = 1,2,..n$ - ratio of spectral density of signal exciting noise (Q) to spectral density of measured error noise (R) for each pair of these noise components (w_j and v_i correspondingly). That index is similar to the observability index χ_i and is helpful for the FBGM analysis in SF mode. Developing the idea of FBGM (1.9), (1.10) a new, more simple KBF suboptimal modification FBGM-Initial/Steady (FBGM-I/S) can be considered. This modification is as follows

$$\dot{\hat{x}} = F\hat{x} + K(z - H\hat{x}),$$
$$K = \begin{cases} K^*_{IF}, & t_0 \le t \le t_*, \\ K^*_{SF}, & t > t_*, \\ 0 & if \quad z \equiv 0 \end{cases} \qquad (1.11)$$

where K^*_{IF} and K^*_{SF} both are constant filter matrix gates chosen for IF and for SF modes correspondingly. (1.11) assumes that $K^*_{IF} >> K^*_{SF}$ and is used during filter transfer process that can be terminated as soon as possible (for a short time) and then the steady filtering process can start and last as long as measurements vector z is available . This idea just reflects conventional engineering approach for linear control system design to extend system bandwidth at the transfer process period and to narrow it at the steady state work period. However, if it is clear that the steady state matrix-gate K^*_{SF} in (11) can be calculated using KBF formulas for K^* (1.10) (3-rd and 4-th). The question about how to determine the gate K^*_{IF} can be discussed additionally, but this gate is usually beggar then K^*_{SF}, providing fast decaying of the transfer process caused by the initial conditions.
Indeed, this matrix gate can be just designated using conventional engineering criteria such as stability margin, overshooting, decaying time, considering filter characteristic polynomial

$$\Delta(s) = sI - (F + kH) = s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_0 \qquad (1.12)$$

and arranging its coefficients a_i to provide to (12) desired roots. For example, standard coefficients for the multiple roots λ

$$\Delta(s) = (s + \lambda)^n \qquad (1.13)$$

where λ determines system cut frequency (bandwidth) that is inverse to its response time and is usually limited by the maximum available static control action. Then matrix K^*_{SF} can be determined considering desired coefficients a_i (1.12) or the bandwidth λ and standard coefficients (1.13).

However, it is important to note that the reliable information about matrixes Q and R is usually not available almost at any stage of system development and operation. Especially, this is related to the matrix Q. Hence, the question about the reliability of optimal filter gain K_{SF}^{*} is also appears and can be discussed. In this case, when matrix Q is not available, it can be accepted as zero $Q = 0$ and P^{*} is not optimal, but at some satisfactory level $P^{*} = D$. Then K_{SF}^{*} can be found from the equation

$$F^{*}D + P^{*}D + K_{SF}^{*}HRH^{T}K_{SF}^{*T} = 0 \qquad (1.14)$$

where $F^{*} = F - K_{SF}^{*}H$,

This equation is covariance equation for the filter estimation errors caused by the measuring noise $v(t)$

$$\dot{\tilde{x}} = F^{*}\tilde{x} - K_{SF}^{*}Hv \qquad (1.15)$$

In fact, in practice the real physical processes $w(t)$ and $v(t)$ have more complex than Gaussian stationary white noise structure and hardly can be expressed by the matrixes Q and R, assumed in KBF theory. However, this abstraction could be helpful for practical needs if some “appropriate” levels of Q and R are taken for the AD that would be resulted in solution, which is compatible with conventional engineering practice.

2.**Using Kalman Filter for the closed loop feedback control purposes**

2.1 State estimator plus controller

As it follows from the System Control and Estimation theory optimal control and estimation can be achieved in the negative feedback control loop with the consecutive connection of optimal controller and optimal estimator. If system is linear and the optimization criteria are minimum of a quadratic functional for both control and estimation processes, then optimal estimator and controller gains can be found from the solution of similar matrix Riccati equations that are *dual*.

In other words, this dualism allows for solving both problems with a single unit, KBF (FBGM) – estimator used in the feedback close control loop as a controller that makes closed system dynamic equivalent to the dynamic of open loop estimator’ errors.

In general, two units are needed; system state estimator (KBF/FBGM) and some kind of the controller, that can be optimal or not, but performing required control functions to bring the system plant to the required state and maintain it counteracting to the acting disturbances. Such a system can be represented by the block diagram in Figure 1

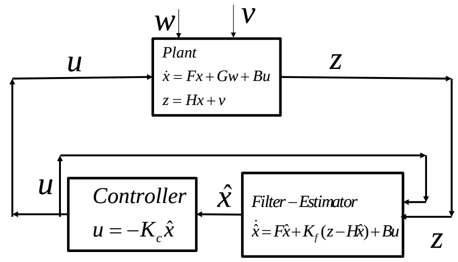


Figure 1 Closed loop with estimator and controller in negative feedback closed loop.
Matrix mathematical equation for this diagram is presented below

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} F & -BK_c \\ K_f H & F - K_f H - BK_c \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} Gw \\ K_f v \end{bmatrix} \qquad (2.1.1)$$

where K_c is the controller matrix control gain, B is control vector u input matrix.
The determinate of (2.1.1) is as follows

$$\Delta = \det \begin{bmatrix} sI - F & BK_c \\ -K_f H & sI - F + K_f H + BK_c \end{bmatrix} \qquad (2.1.2)$$

The optimal controller and the estimator can be designed independently and then used in the consecutive negative feed-back control loop that would stable and meet optimization criteria. This statement is known as *the Separation Theorem.* Let us introduce estimation error

$$\tilde{x} = x - \hat{x} \qquad (2.1.3)$$

Then (18) can be transformed to the following form

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} F - BK_c & BK_c \\ 0 & F - K_f H \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} Gw \\ Gw - K_f v \end{bmatrix} \qquad (2.1.4)$$

That shows that system determinant is

$$\Delta = \det \begin{bmatrix} sI - F + BK_c & -BK_c \\ 0 & sI - F + K_f H \end{bmatrix} = \det[sI - (F - K_f H)] \cdot \det[sI - (F - BK_c)] = p(s) =$$

$$= p_f(s) \cdot p_c(s) = (s^{n/2} + a_{n/2-1} s^{n/2-1} + .. + a_0) \cdot (s^{n/2} + b_{n/2-1} s^{n/2-1} + .. + b_0) \qquad (2.1.5)$$

where

$$p_f(s) = \det[sI - (F - K_f H)] = s^{n/2} + a_{n/2-1}s^{n/2-1} + .. + a_0,$$
$$p_c(s) = \det[sI - (F - BK_c)] = s^{n/2} + b_{n/2-1}s^{n/2-1} + .. + b_0$$

In (Braison &Yu-Chi Ho, 1975) are presented equations for choosing K_c that would provide minimum to quadratic control quality criteria. However, K_c can also be chosen using conventional engineer approaches for negative feedback closed control loop design, providing permissible stability margin, decaying time and overshooting. As well the filter coefficients (matrix K_f) can also be chosen from the similar considerations.

2.2 State estimator as controller

If the same quality criterion (1.4) is used for both estimation and control processes, then as it follows from the *Dual Principle,* there is no need for both units (state estimator and controller). The problem can be solved by a single estimator implemented in the system as the negative closed loop feedback. That would provide the same feedback closed loop control system errors (standard deviations –STD) as the open loop state estimator for estimation errors. Or in other words, this closed loop control system models the open loop state estimator.

This approach presumes that all used in control law state vector components ($\tilde{x}_i$) are directly measured or can be derived from the measured signal.
Indeed, let us introduce estimation error $\tilde{x}$

$$\tilde{x} = x - \hat{x} \qquad (2.2.1)$$

Then subtracting second raw of matrix equation (16) from the first raw we can get following matrix equation

$$\dot{\tilde{x}} = \tilde{F}\tilde{x} + Gw - K_f v \qquad (2.2.2)$$

where Let us introduce estimation error $\tilde{x}$

$$\tilde{x} = x - \hat{x} \qquad (2.2.3)$$

Then subtracting the Filter equation (Filter block) from the Plant equation in Figure 1 we can get the equation of close loop controlled system errors (as in the second raw of (2.1.14))

$$\dot{\tilde{x}} = \tilde{F}\tilde{x} + Gw - K_f v \qquad (2.2.4)$$

where $\tilde{F} = F - K_f H$ and K_f is determined from (1.5). System with state estimator performing estimation and control tasks in the closed loop is presented in Figure 2.

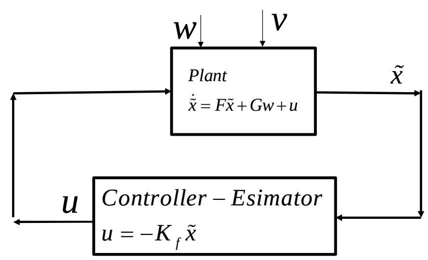


Figure 2 Sate estimator (filter) as system closed loop controller

2.3 External multisensory state estimator and controller

In some cases system space vector x components can be measured by more than one sensor. Few sensors using different physical principles and having different frequency spectrum of errors can be applied, providing to the system rather informational, than reliability redundancy.

In this case system state estimator can be made invariant to the measured signal (x) and estimating measured errors rather than system state.

Let us assume that, at least, two sensors are measured system state (same vector components). One sensor (b) is basic and has low frequency band spectrum errors that grow with time and second one (c) has high frequency band spectrum error and is used to correct errors of the first sensor. These sensors are united in joined Multisensory Sensor Unit (MSU) by means of using optimal estimator to estimate and compensate first sensor errors with calibration it with regard to the second sensor. After the (b) sensor errors compensation measured information becomes the optimal state vector estimate ($\hat{x}$) and is used for system control purposes. In this case the MSU is absolutely uncoupled and independent from the controller. System block diagram is presented in Figure 3.

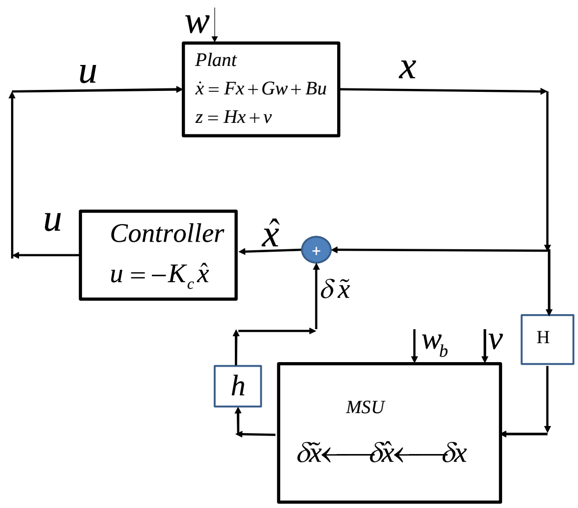


Figure 3 external Multisensory measuring systems (MSU) in the closed loop feedback control

MSU mathematical description is as follows

$$\begin{cases} z_b = H(x + \delta x_b) \\ z_c = H(x + \delta x_c) \\ z = z_b - z_c = H(\delta x_b - \delta x_c) \\ \delta x_c = v \\ \delta \dot{x}_b = F_b \delta x_b + G_b w_b \\ \delta \dot{\hat{x}}_b = F_b \delta \hat{x}_b + K_f (z - H \delta \hat{x}_b) \\ \delta \tilde{x}_b = \delta x_b - \delta \hat{x}_b \\ \delta \tilde{x} = h \delta \tilde{x}_b \end{cases} \qquad (2.3.1)$$

where H is measurement matrix, h is correction matrix, δx_b is vector error of the basic measuring system, δx_c is error of correcting measuring system

These equations are similar to KBF equations (1.1)-(1.5) and all related notations can be referred correspondently.

It can be seen from the Figure 3, that the system state vector x is measured by MSU and has residual error $\delta\tilde{x}$ after estimation $\delta\hat{x}$ and compensation base sensor original error δx. Two sensors are used in the MSU two measure the same signal s that provides the informational redundancy (the signal $s = Hx$ is measured by the two sensors z_1 and z_2). Therefore, the system is invariant regarding to s and estimation and control problems are split in two independent.

The control law coefficients can be chosen considering only desired control performance (tolerant errors caused by the plant disturbance w, stability margin, decaying time, overshooting, and et set.) without taking care about reducing influence of measured sensor errors.

Examples of synthesis of satellite attitude control (Attitude Control system-ACS)

Let us consider a simple second order linear stationary differential equation in the state vector form as below

$$\begin{cases} \dot{x}_1 = w_1 \\ \dot{x}_2 = x_1 \end{cases} \qquad \text{(E.1)}$$

This equation can represent the flat, linear or angular single axis motion of a rigid body in the inertial spatial. Let us assume that this is the angular motion (it could be the linear motion as well) and x_1 is angular velocity, x_2 is angular position, w_1 is the specific external disturbing torque applied to the RB and acting with respect to its CM.

In this case $w_1 = \frac{M_x}{J_x}$ and J_x is the body axial inertia, M_x is disturbing torque. Let us also assume that system (E.1) (plant) state vector x is measured (in general case both components) by some position and angular velocity sensors

$$z = Hx + v \qquad \text{(E.2)}$$

where

$H = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix}$ is measurement matrix, $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is measuremnt error vector

In particular case, h_1 and h_2 can be 1 or 0, or in other words, only position or velocity and both can be measured . Disturbance w_1 and measurement error v in (E.1) and (E.2) are stationary Gaussian white noises having covariance matrixes $Q = \begin{bmatrix} q_1 & 0 \\ 0 & 0 \end{bmatrix}$ and $R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$

In the first approximation system (E.1), (E.2) can represent the simplest mathematical model of satellite single axis attitude control system (ASC), allowing for the principal analysis of system performance with using applied optimal estimation and control theory. Two possible variants are presented below

1- Only satellite angular position is measured

This variant is usually used when mainly only initial satellite body frame alignment to desired reference frame and further stabilization with respect it is required. Two sub-variants are possible here; using KBF is the controller, without special state estimator and using the estimator and the controller jointly as prescribed by the separation theorem.

In this case in satellite model (E.1), (E.2) takes place following $h_1 = 0,\ h_2 = 1$ and $r_1 = 0,\ r_2 \neq 0$, then the estimator equation is as follows

$$\begin{cases} \dot{\hat{x}}_1 = k_{12}(z_2 - \hat{x}_2) \\ \dot{\hat{x}}_2 = \hat{x}_1 + k_{22}(z_2 - \hat{x}_2) \end{cases} \qquad \text{(E.1.1)}$$

Forth equation in (1.10) is matrix algebraic covariance equation for steady state ($P(t \to \infty) = P^*$ /
In this case it allows for analytical solution

$$\begin{cases} P_{12}^* = \sqrt{q_1 r_2} = r_2\sqrt{\xi} \\ P_{22}^* = \sqrt{2 r_2 P_{12}^*} = r_2\sqrt{2\sqrt{\xi}} \\ P_{11}^* = \dfrac{1}{r_2} P_{12}^* P_{22}^* = r_2\sqrt{2\xi\sqrt{\xi}} \end{cases} \qquad \text{(E.1.2)}$$

where $\xi = \dfrac{q_1}{r_2}$ is *filterability* ratio

Filter coefficients (gates) are

$$\begin{cases} k_{12} = \dfrac{P_{12}^*}{r_2} = \sqrt{\xi} \\ k_{22} = \dfrac{P_{22}^*}{r_2} = \sqrt{2\xi\sqrt{\xi}} \end{cases} \qquad \text{(E.1.3)}$$

Filter equation (E1.1) can be rewritten in the conventional (automatic control theory) form for 2-nd order differential equation

$$T^2\ddot{\hat{x}}_2 + 2dT\dot{\hat{x}}_2 + \hat{x}_2 = T^2\dot{z}_2 + 2dTz_2 \qquad \text{(E.1.4)}$$

where $T = \dfrac{1}{\sqrt{k_{12}}} = \dfrac{1}{\sqrt[4]{\xi}}$ is filter time constant, $d = \dfrac{k_{22}}{2\sqrt{k_{12}}} = \dfrac{\sqrt{2}}{2} = 0.707$ is filter specific damping coefficient.

As it can be seen from (E.1.2) estimation errors are proportional to the filterability ratio ξ and filter time constant is in inverse proportionality to this ratio or in other words small ξ requires a big time to filter out signal from noise and vise versa.
This is a common feature of any liner filter. Damping coefficient in this case $d = 0.707$ (optimal value for minimum decaying time) of any transfer process in (E.1.4) independently of system stochastic characteristics (matrix Q and R).

1.1 Using filter/estimator as the controller

In this case (E.1.1) as in (2.2.2) can be implemented, the same closed feedback control loop is used as the estimator and the controller simultaneously. Indeed, if minimum diagonal P^* is common quality criterion for estimation and control of (E.1), then subtracting (E.1) from (E.1.1) we can get equation of estimated errors

$$\begin{cases} \dot{\tilde{x}}_1 = -k_{12}\tilde{x}_2 - k_{12}v_2 + w_1 \\ \dot{\tilde{x}}_2 = \tilde{x}_1 - k_{22}\delta\tilde{x}_2 - k_{22}v_2 \end{cases} \qquad \text{(E.1.1.1)}$$

that can be interpreted as the negative feedback closed control loop

$$\ddot{\tilde{x}}_2 = -\frac{1}{T^2}\tilde{x}_2 - 2\frac{d}{T}\dot{\tilde{x}}_2 - \frac{1}{T^2}v_2 - 2\frac{d}{T}\dot{v}_2 + w_1 \qquad \text{(E.1.1.2)}$$

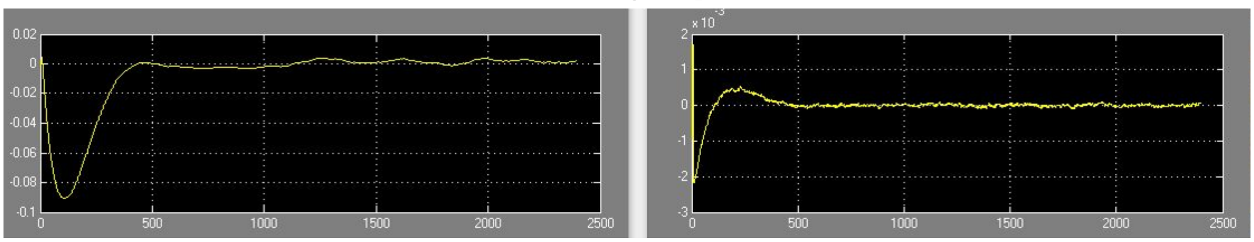
where $T = \frac{1}{\sqrt{k_{12}}}, d = \frac{k_{22}}{2\sqrt{k_{12}}}$

(E.1.1.2) was simulated with coefficients calculated with formulas (E.1.2), (1.1.3) that provide corresponding time constant T and damping coefficient $d = 0.707$.
Despite this variant doesn’t use special state estimator, however it assumes that angular velocity will be derived differentiating measured attitude. This differentiation with smoothing of the amplified, due to the differentiation measured noise, can be considered as an angular velocity estimator also.

Were used following numerical data:
$J = 50\ \text{kgm}^2$ is satellite inertia, $\sigma_M = 10^{-6}\ \text{Nm}$ is satellite disturbing torque (noise) standard deviation (STD), $T_M = 10\ \text{s}$ is disturbing torque correlation time, $\sigma_{v2} = 0.1\ \text{deg}$ is satellite attitude measured error (noise) STD, $T_{v2} = 0.1\ \text{s}$ is satellite attitude error correlation time.
Noise covariances can be calculated as for correlated stochastic processes (limited frequency band white noise) with *correlation time** T_M and T_{v_2} accordingly $q_1 = \frac{2\sigma_M^2 T_m}{J_x^2} = 8\cdot 10^{-14}\ \text{rad}^2/\text{s}^3$ and $r_2 = 2\sigma_{v2}^2 T_{v2} = 6.092\cdot 10^{-7}\ \text{rad}^2\cdot\text{s}$.
Calculated optimal filter steady state parameters: optimal coefficients $k_{12} = 0.0001146\ 1/\text{s}^2$, $k_{22} = 0.01514\ 1/\text{s}$, time constant $T = 93.42\ \text{s}$ (for SF mode) and, specific damping $d = 0,707$, STD estimation errors: $\sigma_1 = 5.9\cdot 10^{-5}\ \text{deg}/s$, $\sigma_2 = 5.5\cdot 10^{-3}\ \text{deg}$.
For the IF mode time constant was taken $T_{IF} = 3\ \text{s}$, $d = 0,707$.

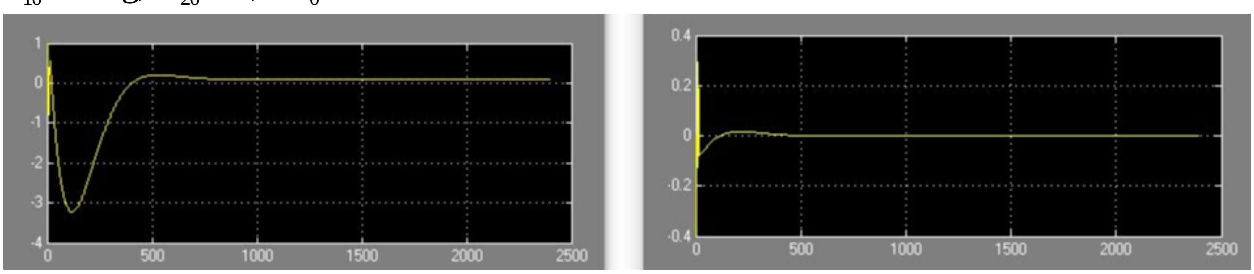
Additional constant disturbance torque $\sigma_{M_0} = 10^{-5}$ Nm (uncounted in KBF model) was applied to the satellite to demonstrate effect of its influence on satellite attitude control process with KBF (FBGM).
Simulation results are presented in Figure 4. Angular velocity $\tilde{x}_1$ was derived as the derivative from the measured attitude signal $\tilde{x}_2$ using differentiating filter $D(s) = \dfrac{s}{\tau s + 1} (\tau = 5 \text{ s})$ for smoothing differentiated noise . Time of switch FBGM from IF to SF mode is $t^* = 15$ s

1-Zero initial conditions and additional disturbing torque: $x_{10} = 0,\ x_{20} = 0,\ M_0 = 0$



Attitude error (deg) Attitude velocity error (deg/s)

2-Non zero initial angular deviation and additional disturbing torque:
$x_{10} = 1\text{deg},\ x_{20} = 0,\ M_0 = 10^{-5}\text{Nm}$



Attitude error (deg) Attitude velocity error (deg/s)
Static attitude error is $\tilde{x}_2^* = 0.1\text{deg}$
Figure 4 Only attitude is measured. Using Filter as a Controller.

*Note: This presumes that synthesized filter time constant is much bigger than these noise time constants ($T_M << T$ and $T_{v_2} << T$).

The main advantage of this scheme is its simplicity. It has a constrains that following inequality should be satisfied $\tau << T$ and a single parameter T should provide simultaneously contradictive capacities: filtering noise, fast decaying the transient process and compensate disturbing torque with a small static error.

1.2 Using filter/estimator and controller

In this case both , estimator and controller are used as in (2.2.2), being consequently connected in closed feedback control loop. The same numerical data and the same filter (E.1.3), as in the example above, were used. However, special controller, as in (2.2.2), was added in the feedback closed control loop for the control purposes

$$u = -k_d \hat{x}_1 - k_p \hat{x}_2 \qquad \text{(E.1.2.1)}$$

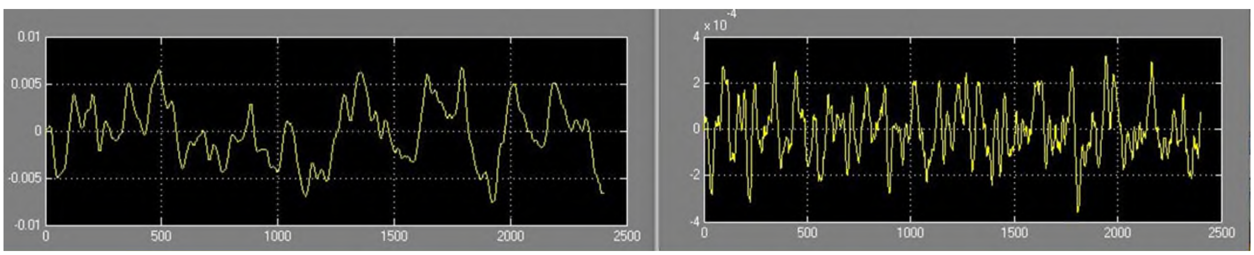
where k_d is damping coefficient and k_p is position coefficient that provide angular velocity and positioned control to (E.1), based on state estimates $\hat{x}_1$ and $\hat{x}_2$.

These control coefficients correspond to certain time constant $T_c = \sqrt{\frac{k_p}{J}}$ and specific damping coefficient $d_c = \frac{k_d}{2\sqrt{k_p J}}$. Not optimal, but following numerical values were chosen for the control parameters $d_c = 0.707,\ T_c = 10$ s.

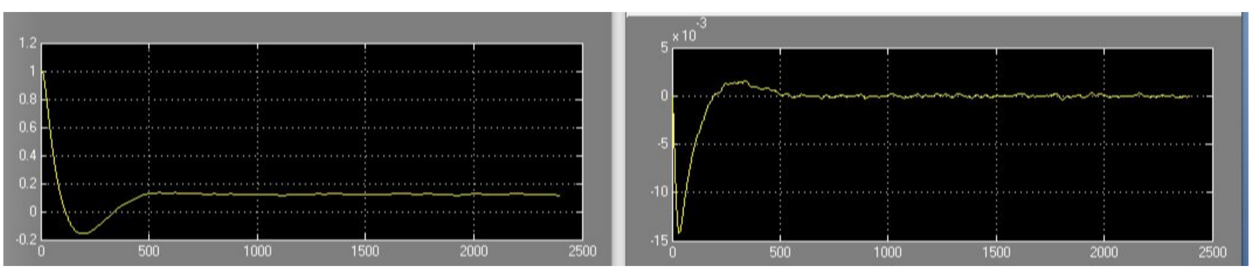
Time of FBGM switch from IF to SF mode is $t^* = 9$ s, Time constants: $T_{SF} = 93.4$ s, $T_{IF} = 3$ s, filter specific damping coefficient $d = 0.707$.

Simulation results are presented in Figure 5.

1-Zero initial conditions and additional disturbing torque: $x_{10} = 0,\ x_{20} = 0,\ M_0 = 0$



2-Non zero initial attitude deviation and additional disturbing torque:
$x_{10} = 1\text{deg},\ x_{20} = 0,\ M_0 = 10^{-5}\text{Nm}$



Static attitude error is $\tilde{x}_2^* = 0.1\text{deg}$

<u>Figure 5.</u> Only attitude is measured, control with optimal estimator and controller

This scheme is more flexible T_F can be responsible for filtering measured noise and T_C -for the time response and static error, caused by the disturbing torque.

2-Satellite angular position and velocity are measured

If satellite attitude control tasks includes not only stabilisation, but also angular maneuvering (slew), then angular velocity sensor is appropriate to use in addition to attitude sensor.

Let us assume that both satellite state vector components (x_1 -velocity) and (x_2 -position) are measured with the velocity sensor (S_1) and the angular sensor (S_2), having Gaussian white noise errors v_1 and v_2 correspondingly. All the numerical data are as in above example, but with addition of v_1 . That has following noisy error with STD $\sigma_{v1} = 0.01\,\text{deg/s}$ and correlation time $T_{v1} = 0.1\,\text{s}$.

In this case algebraic equation Riccati , 4-th equation in (1.10) cannot be solved analytically. The numerical solution that was found gives the following parameters:

$\sigma_1 = 0.0008\ \text{deg}/s \quad \sigma_2 = 0.05\,\text{deg}$ are STD of estimation errors and

$k_{11} = 0.0054$, $k_{12} = 0.000272, \quad k_{21} = 0.272, \quad k_{22} = 0.0236$ are filter coefficients. The filter coefficients can be converted in its time constant $T = \dfrac{1}{\sqrt{k_{11}k_{22} - k_{12}k_{21} + k_{12}}} = 55.3\ \text{s}$ and specific damping coefficient $d = \dfrac{k_{11} + k_{22}}{2T} = 2.62 \cdot 10^{-4}$. It can be seen that calculated optimal damping coefficient is too small to provide in this estimator free oscillation decaying and this optimal KF cannot be practically used.

Therefore was used different approach; the coefficient d was set as $d = 0.707$, but chosen optimal $T = 55.3\ \text{s}$.

There were added some additional factors to introduce some differences with KBF idealistic model ; constant disturbing torque $\sigma_{M_0} = 10^{-5}\ \text{Nm}$ (as above) and constant angular velocity measurement error (bias) $\Delta_{x1} = 10\,\text{deg}/h = 0.0028\,\text{deg}/s$. For IF mode was taken that: $T_{IF} = 3\,\text{s}$, $d = 0.707$ and $t^* = 9\ \text{s}$ .Two variants were analysed as for the case of measuring attitude only above

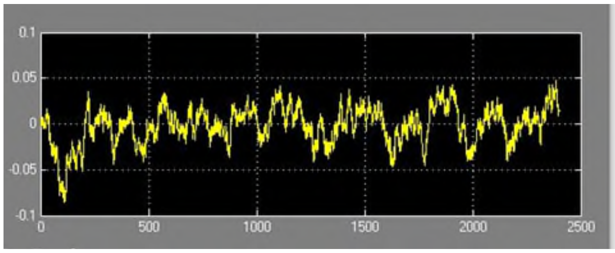
2.1 Using filter/estimator as the controller

The same simulation scheme as above was used for the simulation. Measurement of angular velocity z_1 was added with the noisy error v_1 and the bias Δ_{x1} and this measurement was used instead of the differentiator in addition to angular measurement z_2

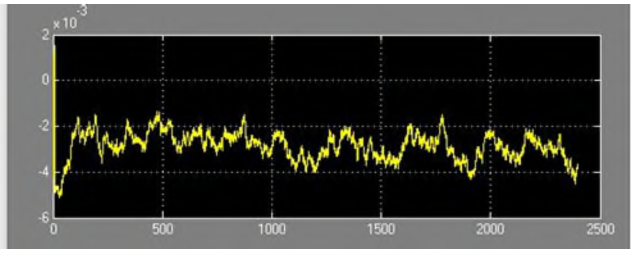
The simulation results are presented in the Figure 6.

1-Zero initial conditions and additional disturbing torque: $x_{10} = 0, \ x_{20} = 0, \ M_0 = 0$

Time of FBGM switch from IF to SF mode is $t^* = 9\ \text{s}$



Attitude error (deg)

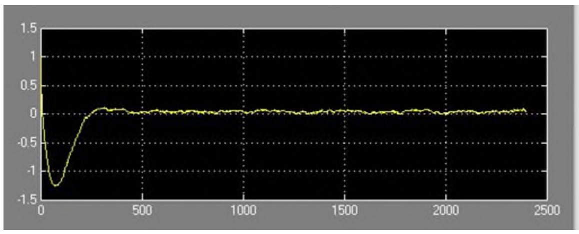


Attitude velocity error (deg/s)

Static velocity error is $\tilde{x}_1 = -\Delta_{x1} = -0.0028 \deg/s$

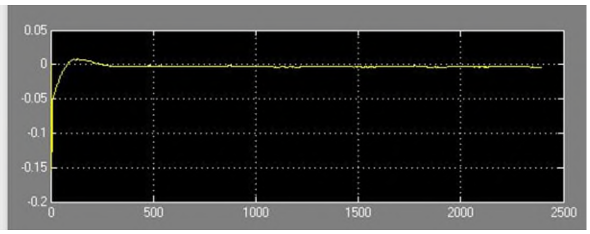
2-Non zero initial attitude deviation and additional disturbing torque:

$x_{10} = 1\deg,\ \ x_{20} = 0,\ \ M_0 = 10^{-5}\,\mathrm{Nm}$



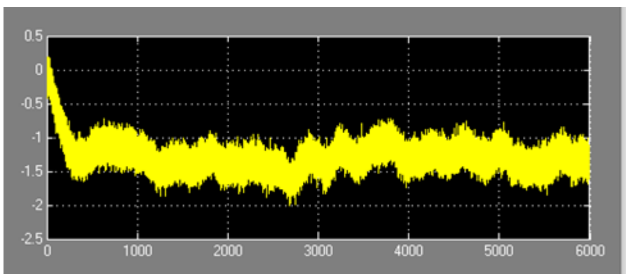
Attitude error (deg)

Static attitude error is $\tilde{x}_2^* = 0.0355 \deg$

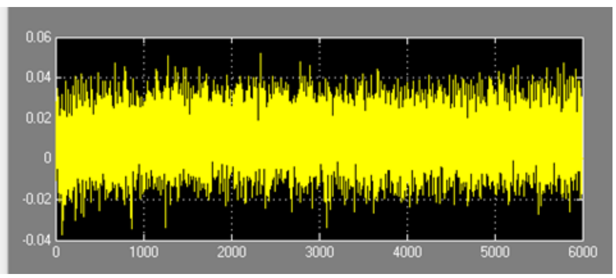


Attitude velocity error (deg/s)

Static velocity error is $\tilde{x}_1 = -\Delta_{x1} = -0.0028 \deg/s$



Angular errors (deg)



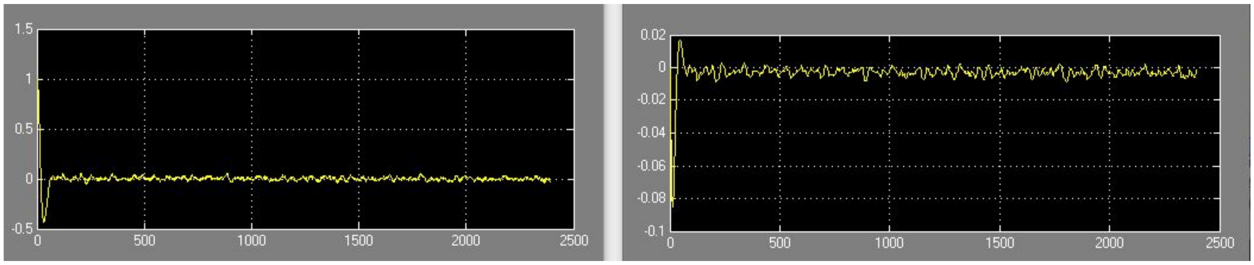
Angular rate errors (deg/s)

Measured (perceived) errors/Sensor outputs)

Figure 6 Attitude and attitude velocity are measured, control with optimal estimator (without controller)

2.2 Using filter/estimator and controller

Similarly to the case 1.2 above the estimator ($T = 53.42$ s, $d = 0.707$) and additional controller were used being consequently connected in the common feedback control loop as in (21.1). The same controller as for the case 1.2 was used ($T_c = 10$ s, $d_c = 0.707$). The simulation results are presented in Figure 7.



Attitude error (deg) Attitude velocity error (deg/s)

Static attitude error is $\tilde{x}_2^* = 0.037\deg$ Static velocity error is $\tilde{x}_1 = -\Delta_{x1} = -0.0028\deg/s$

Figure 7 Attitude and attitude velocity are measured, control with optimal estimator and controller in common feedback closed control loop

2.3 External multisensory measurement unit (MMU)

External measurement unit with informational redundancy (2.3.1) about measured attitude was simulated. The base unit (b) is satellite angular speed sensor (gyro) that output is integrated to get attitude. The corrector (c) is satellite orientation (angular) sensor (for example star tracker). Satellite control law was taken as follows (PD-control)

$$u = -k_d \dot{x}_2 - k_p x_2 \qquad \text{(E.2.3.1)}$$

where $k_d = \frac{2d}{T}$, $k_p = \frac{1}{T^2}$ are specific damping and proportional coefficients

Disturbing torque and measurement errors as follows

$$w = \frac{M}{J},\ M = M_0 + M, v_1 = V_{10} + V_1, v_2 = V_{20} + V_2 - \text{bias plus noise} \qquad \text{(E2.3.2)}$$

where $J = 50 kgm^2$ is satellite moment of inertia,

$M_0 = 10^{-5} Nm, V_{10} = 0.01\ \text{deg/s} = 36\ \deg/h,\ V_{20} = -0.1\deg$ are bias components,

$\sigma_M = 10^{-6}\,\text{Nm},\ T_M = 10s,\ \sigma_{v1} = 0.01\deg/s,\ T_{v1} = 1s,\ \sigma_{v2} = 0.1\deg/s,\ T_{v2} = 0.1s$ are noise components (σ is standard deviation, T is correlation time)

Differential equations of control satellite dynamic is

$$\ddot{x}_2 + k_d \dot{x}_2 + k_p x_2 = w - k_d \delta\tilde{x}_1 - k_p \delta\tilde{x}_2 \qquad \text{(E.2.3.3)}$$

where $\delta\tilde{x}_1$ and $\delta\tilde{x}_2$ are errors of estimates of satellite angular velocity and orientation after their estimation and compensation in MMU as follows $\delta\tilde{x}_1 = \delta x_1 - \delta\hat{x}_1,\ \delta\tilde{x}_2 = \delta x_2 - \delta\hat{x}_2$.

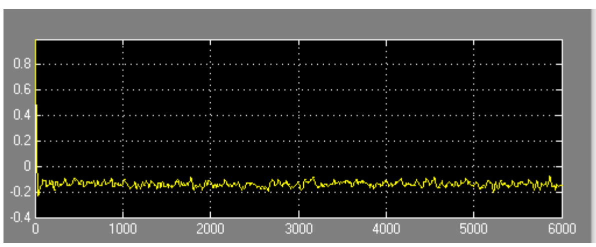
MSMU uses difference between x_2 measured by the basic system $x_{2b} = x_2 + \delta x_2$ where $\delta x_2 = \delta x_{20} + \int_0^t v_1 d\tau$ and by the correcting system $x_{2c} = x_2 + v_2$, , as $z = \delta x_2 - v_2$ and estimates measured by the basic system errors $\delta\hat{x}_1$ and $\delta\hat{x}_2$. After errors compensation, measured angular velocity and position are used for the satellite control. This is resulted in (E.2.3.3). (basic system) bias. Calculated for this scheme filter coefficients k_{12} and k_{22} provide $d = 0.707$ and $T = 3.62$ s This scheme provides estimation and compensation of the velocity sensor bias.

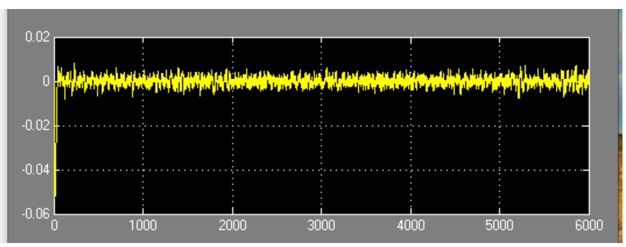
Simulation results are presented in Figure 8.

There were simulated two controllers $T = 10$ s and $T = 100$ s. For both cases $d = 0.707$

1-$T = 10$ s

Without MSMU

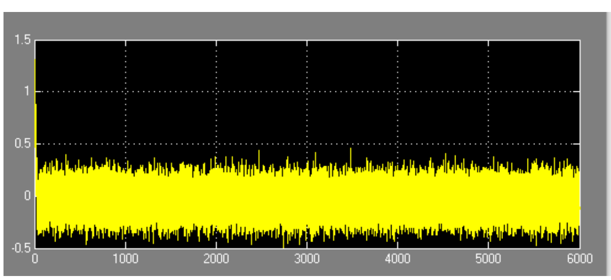


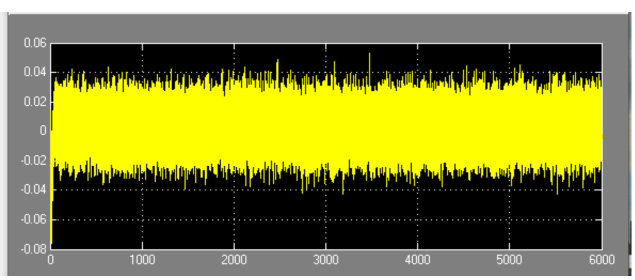


Static error:-0.138 deg

Angular errors (deg) Angular rate errors (deg/s)

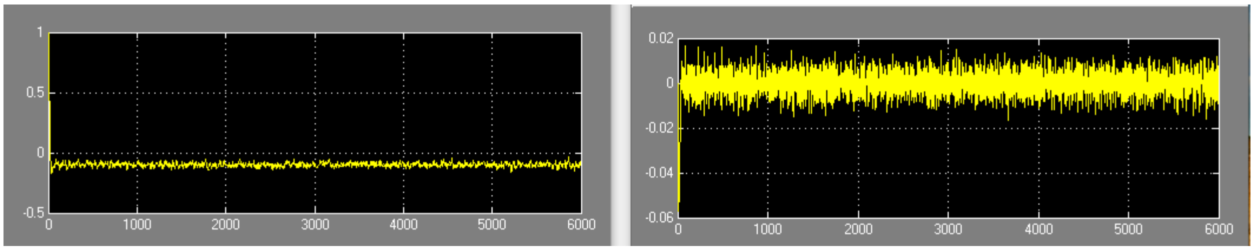
Measured (perceived)





Angular errors (deg) Angular rate errors (deg/s)

With MSMU

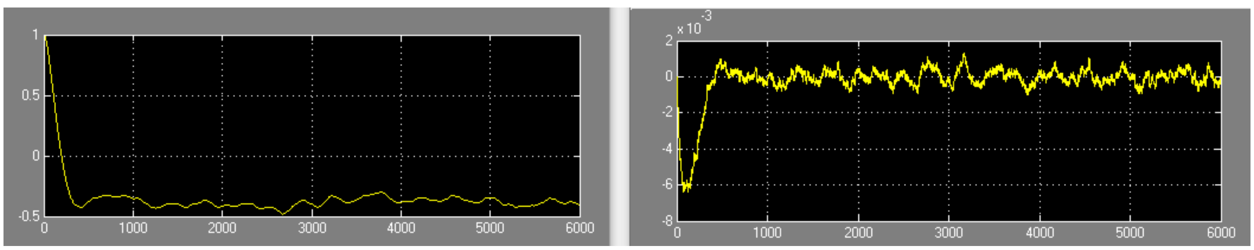


Static error: -0.1 deg=V2o

Angular errors (deg) Angular rate errors (deg/s)

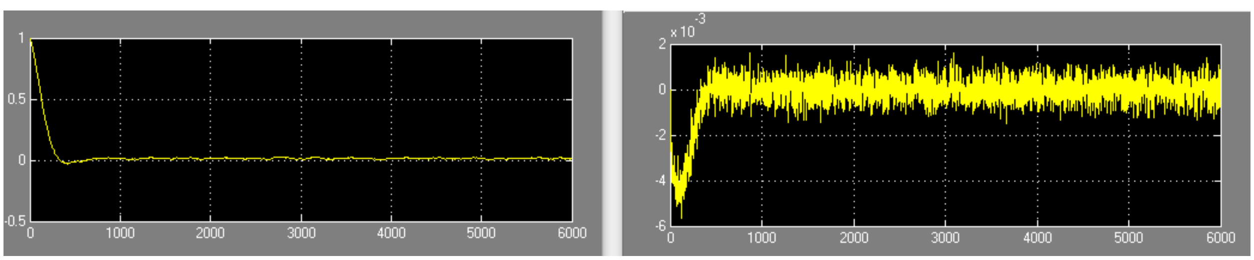
2-$T = 100$ s

Without MSMU



Angel (deg) Static error -0.38 deg Angular velocity (deg/s)

With MSMU



Angel (deg) Angular velocity (deg/s)

Figure 8 Using MSMU for satellite attitude estimation and control

As it can be seen from the simulation results MSMU noticeably improve performance in case when the controller has big time constant to prevent noise influence on attitude control, in this case angular speed sensor bias V_{10} is compensated and doesn't cause additional error (-0.38 deg) in attitude stabilization.

Conclusion

1. Analytical design based on optimal estimation and control methods is helpful approach to verify control system feasibility and predict potentially available performance.

2. Direct formal implementation of KF with a rigorous recursive solution of the Riccati equation and computation of the corresponding variable control gains can be resulted in practically useless wasting of OBC resources and even in not asymptotically stable filter behavior.
3. Suboptimal KF-steady state FBGM in many cases can provide a satisfactory solution being stable and robust and can be used as the estimator and the controller simultaneously
4. Informational redundant, conventional in Aviation MSMU can be successfully used for a spacecraft control purposes, that can bring to the design additional benefits.

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